



# Dynamically possible pattern speeds of double bars

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**Abstract.** The method to study oscillating potentials of double bars, based on invariant loops, is introduced here in a new way, intended to be more intelligible. Using this method, I show how the orbital structure of a double-barred galaxy (nested bars) changes with the variation of nuclear bar's pattern speed. Not all pattern speeds are allowed when the inner bar rotates in the same direction as the outer bar. Below certain minimum pattern speed orbital support for the inner bar abruptly disappears, while high values of this speed lead to loops that are increasingly round. For values between these two extremes, loops supporting the inner bar extend further out as its pattern speed decreases, and they become more eccentric and pulsate more. These findings do not apply to counter-rotating inner bars.

**Key words.** stellar dynamics — galaxies: kinematics and dynamics — galaxies: nuclei — galaxies: spiral — galaxies: structure

## 1. Introduction

Double-barred galaxies are barred galaxies, where a second, smaller bar is nested inside the larger bar (see Erwin, this volume, for the review). Two independent surveys (Erwin & Sparke 2002; Laine et al. 2002) indicate that up to 30% of barred galaxies host nested bars, but cross-correlation of these samples implies that this percentage may be lower (Moiseev, this volume). Observed random orientation of the two bars in double-barred galaxies indicates that the bars may rotate independently. This was confirmed for NGC 2950, where Tremaine-Weinberg integrals are inconsistent with a single rotating pattern (Corsini et al. 2003).

Our understanding of the dynamics of barred galaxies strongly relies on studies of periodic orbits. Maciejewski & Sparke (1997,

2000) showed that similar studies can be pursued for double bars. Closed periodic orbits in a single bar correspond to double-frequency orbits in double bars (Maciejewski & Athanassoula 2007, 2008). These orbits can be studied through their maps called loops or invariant loops. In Section 2, we show the benefit of studying double-frequency orbits with this method. In Section 3, we apply this method to study how the structure of the inner bar depends on its pattern speed, and whether all pattern speeds of that bar are dynamically possible.

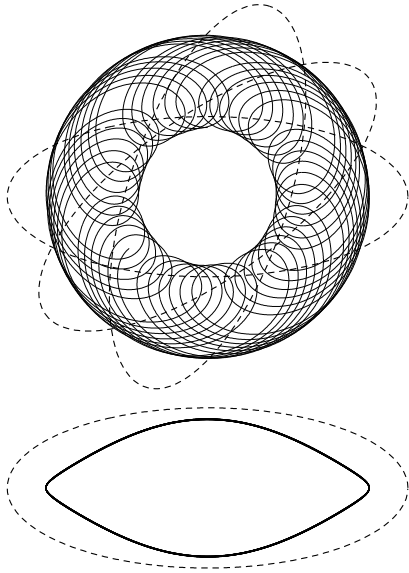
## 2. Why study double bars with loops

In orbital studies one assumes the form of the potential (which can vary with time), and calculates orbits in this potential. A single bar is usually treated as a fixed, rigidly rotating potential. Stable closed periodic orbits form the

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backbone of the bar, and their shapes and extent provide information about the structure of the bar. However, these orbits close only in the frame rotating with the bar; in other frames they have a rosette-like appearance (Fig. 1), which no longer displays the information that the orbit can give about the structure of the bar. Moreover, if we want to relate the appearance of the orbit to the structure of the bar, we should study this orbit in a frame in which the potential does not change (is stationary). Otherwise every moment on the orbit would correspond to a different shape of the potential, and one would not be able to relate the shape of the orbit to the shape of the potential.



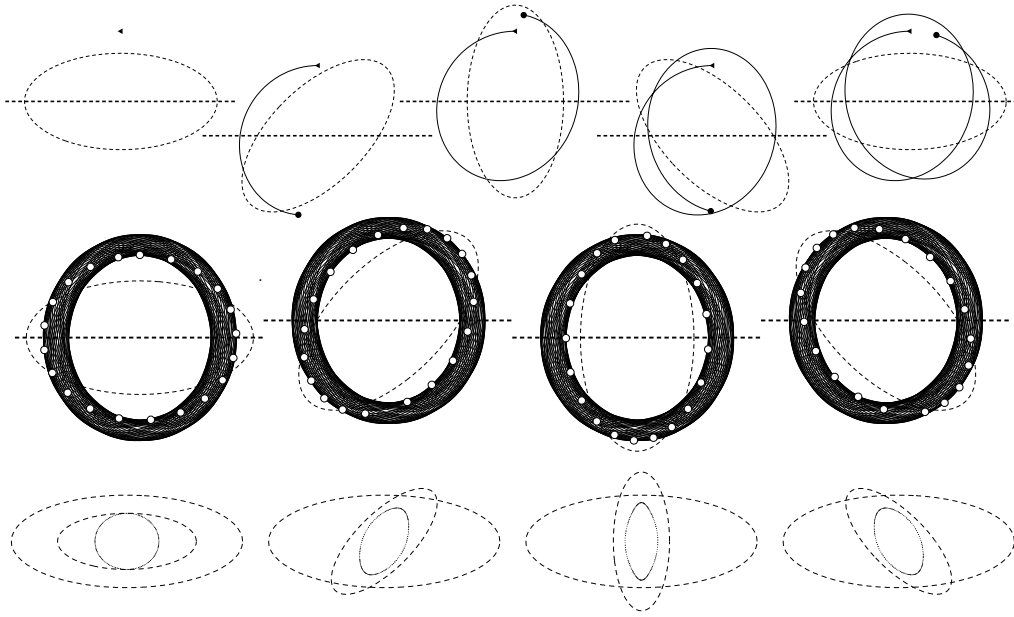
**Fig. 1.** Stable closed periodic orbit in a single bar (an  $x_1$  orbit) in the frame where the bar rotates (upper panel) and in the frame rotating with the bar (lower panel). The dashed line outlines the bar. Only in the lower panel the potential does not change with time (is stationary).

Maciejewski & Athanassoula (2007) showed that closed periodic orbits in a single bar correspond to double-frequency orbits in double bars. This makes orbital studies of double bars more complicated, because double-frequency orbits do not close.

Moreover, if the two bars rotate independently, then the potential changes with time, and relating the shape of the orbit to the shape of the potential, as we could do in the case of a single bar above, is no longer possible. However, as we explain below, we can still relate any particular shape that the periodically changing potential of double bars takes to a sample of points from the orbit taken at moments when the potential has this given shape.

In the top row of Fig. 2, an example double-frequency orbit in double bars is followed as the bars rotate through each other, and drawn in the frame in which the outer bar remains horizontal (both the rotation of the inner bar and the motion on the orbit are counterclockwise). In a sequence of panels from left to right, the orbit starts when the bars are aligned at the location marked by a triangle, develops as the bars get out of alignment, until they align again. In each panel, the relative orientation of the bars is shown for the moment when the position on the orbit is marked by the round dot. The crucial observation that underlies the method presented here is that out of the segment of the orbit presented in each panel, *only this dot is relevant to the shape of the potential shown there*. All other points on the presented segment of the orbit are relevant to other shapes of the potential at other times. As the bars re-align (right-most panel), the orbit does not close, but out of its segment drawn there, two points are now relevant to the shape of the potential presented there: the triangle and the round dot.

Since double-frequency orbits do not close, they contain infinite sets of points relevant to any shape that the oscillating potential can take. In the middle row of Fig. 2, we show a segment of the same double-frequency orbit as in the top row, but now followed for 20 alignments of the bars. At each panel, round dots mark points sampled from this orbit at the moments when the potential takes the shape outlined in a given panel. These points appear to fall on a closed curve, as confirmed in the bottom row of Fig. 2, where for clarity we omitted the double-frequency orbit, and only plotted points sampled from its segment span-



**Fig. 2.** **Top:** Double-frequency orbit in double bars followed from one to the next alignment of the bars. Triangle marks the starting point and the direction of the orbit. Dashed straight horizontal line is drawn along the major axis of the outer bar, with which the frame rotates. The inner bar is outlined with the dashed line. Round dots mark the point on the orbit at the relative position of the bars outlined. **Middle:** Same as in the top row, but for the orbit followed for 20 alignments of the bars. **Bottom:** Points on a double-frequency orbit selected only at the moments when the relative position of the bars is as outlined with dashed lines. These points constitute the loop.

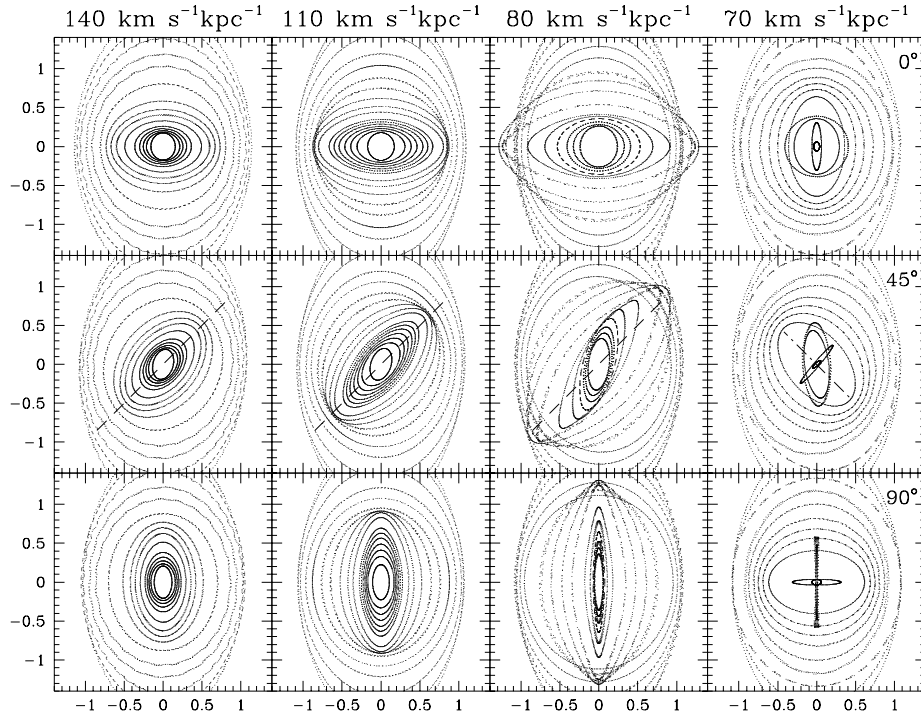
ning 60 alignments of the bars. These points constitute the loop: if a particle is on a non-closing double-frequency orbit, then at any given shape of the periodically changing potential, it will be located somewhere on the loop. Thus representing double-frequency orbits with loops allows us to relate them to any instantaneous shape of the potential, even though the potential changes with time and the orbit does not close.

It is important to stress here that loop is not an orbit, but it is a representation of the orbit, or a sample of relevant points from this orbit. Loops are unrelated to loop orbits with whom they are sometimes confused. Orbital analysis is much faster than constructing  $N$ -body models of double bars, and since parameters of the bars can be changed arbitrarily, it can explore any range of parameters of double bars. Maciejewski & Athanassoula (2008) analyzed extent of double-frequency orbits in 23

models of double bars, whose parameters were varied. They noticed that the trapping of trajectories around double-frequency orbits strongly depends on the inner bar's pattern speed. The phase-space volume occupied by trapped orbits monotonically increases with that pattern speed, accompanied by decreasing chaos. This is contrary to previous expectations that resonant coupling between rotating patterns should minimize chaos (Sygnet et al. 1988).

### 3. How orbital support for inner bar changes with its pattern speed

In a (singly) barred galaxy with an inner Lindblad resonance (ILR), there are two major orbital families: the  $x_1$  orbits, elongated along the major axis of the bar, and the  $x_2$  orbits, elongated perpendicularly to it. If within the ILR, there is another, independently rotating secondary bar (a double-barred galaxy), then



**Fig. 3.** Representative  $x_2$  loops (i.e. loops that correspond to the  $x_2$  orbits in a single outer bar) for four orbital models of double bars from Maciejewski & Small (in prep.), drawn in a frame in which the outer bar remains horizontal. Each column presents a different model, with the pattern speed of the inner bar in that model given at the top of the column. In the top row, the loops are drawn for the moment when the angle between the bars is  $0^\circ$ , in the middle row when it is  $45^\circ$ , while in the bottom row when this angle is  $90^\circ$  (as marked in the right-hand column). In the middle row, the major axis of the inner bar in the imposed gravitational potential is drawn with a dashed line, except for the rightmost panel, where the minor axis of the inner bar is drawn with a dash-dot line. Units on axes are in kpc.

the loops corresponding to the  $x_1$  orbits in the outer bar (i.e. the  $x_1$  loops) remain elongated with that bar (Maciejewski & Sparke 2000). If the secondary bar rotates in the same direction as the outer bar, then among the  $x_2$  loops (defined as those that correspond to the  $x_2$  orbits in the outer bar) the outer ones remain perpendicular to the outer bar, but the inner ones align with the inner bar. *Note that in this notation, when the two bars rotate in the same direction, loops that support the inner bar are the  $x_2$  loops – they correspond to the  $x_1$  orbits in the inner bar.* When the two bars rotate in opposite directions, loops that support the inner bar originate from the  $x_4$  orbits of the outer bar (Maciejewski 2008).

Maciejewski & Small (in prep.) studied the  $x_2$  loops in seven models of double bars that rotate in the same direction with the pattern speed of the inner bar being set at 70 to 140  $\text{km s}^{-1} \text{kpc}^{-1}$ , depending on the model. They analyzed how the orbital support of the inner bar changes with its pattern speed. Parameters for five models were taken from models 01-05 of Maciejewski & Athanassoula (2008), hence for everything except for the inner bar's pattern speed, they have the values of Model 1 from Maciejewski & Sparke (2000).

Representative  $x_2$  loops for four out of seven models analyzed by Maciejewski & Small (in prep.) are shown in Fig. 3. One

can immediately notice several trends, further quantified by Maciejewski & Small:

1. The orbital model by Maciejewski & Sparke (2000) indicates that the inner bar should end well within its corotation, which was then confirmed by  $N$ -body simulations of double bars (Debattista & Shen 2007). In Fig. 3 we see that the orbital support for the inner bar extends further out in radius for lower pattern speeds. However, lower pattern speed means larger corotation radius, and the ratio of the extent of orbital support of the inner bar to its corotation radius remains remarkably constant for the models considered here. This implies that the inner bar can extend to  $(40 \pm 2)\%$  of its corotation.
2. However, one can notice that for orbits that support outer parts of the inner bar when its pattern speed is  $80 \text{ km s}^{-1} \text{ kpc}^{-1}$ , the sampled points are slightly scattered around the expected closed curves. This reflects the general trend for lower pattern speeds that double-frequency orbits supporting outer parts of the inner bar do not trap large volumes of phase-space, hence provide only limited support for the bar.
3. The inner bar pulsates as it rotates through the outer bar, as found by Maciejewski & Sparke (2000) and confirmed by  $N$ -body simulations. From Fig. 3 one can see that as the pattern speed of the inner bar decreases, loops that support it become more eccentric and pulsate more as the bars rotate through each other.
4. As originally noticed by Maciejewski & Sparke (2000) and confirmed by  $N$ -body simulations, loops that support the inner bar overtake the figure of the bar in the imposed potential (its major axis drawn with dashed line in Fig. 3) when the bars get out of alignment. The magnitude of this effect is virtually constant at higher pattern speeds: when the angle between the bars in the imposed potential is  $45^\circ$ , loops supporting the inner bar lead the figure of that bar by  $6^\circ \pm 0.5^\circ$ . Also, the loops rotate coherently at higher pattern speeds, i.e. their major axes are aligned to within a few degrees.

On the other hand, for pattern speeds  $90 \text{ km s}^{-1} \text{ kpc}^{-1}$  and lower, the angle by which the loops lead the bar increases, and the major axes of the loops are no longer aligned.

5. The pattern speed of the inner bar cannot be arbitrarily low. As can be seen in the right-hand column of Fig. 3, when this pattern speed drops from  $80$  to  $70 \text{ km s}^{-1} \text{ kpc}^{-1}$ , loops that support the inner bar are completely wiped out. They are replaced by loops, whose orientation changes in relation to the inner bar (represented by four innermost loops), or remains perpendicular to that bar, indicating that they may be related to the  $x_2$  orbits in the *inner* bar. Interestingly, we found no models in which loops corresponding to the  $x_1$  and  $x_2$  orbits in the inner bar coexist. As can be seen in fig.9 of Maciejewski & Athanassoula (2008), in linear approximation loops corresponding to the  $x_2$  orbits in the inner bar should appear already for pattern speeds of  $100 \text{ km s}^{-1} \text{ kpc}^{-1}$  and below, but they appear only at  $70 \text{ km s}^{-1} \text{ kpc}^{-1}$ , and their appearance is accompanied by vanishing of orbits that support the inner bar.

In short, a given secondary inner bar in a double-barred galaxy can rotate at a rate within a limited range. The lower limit is set by an abrupt destruction of orbits that support it, while the soft upper limit comes from the bar becoming increasingly rounder and being no longer a distinct dynamical feature as its pattern speed increases. These limits apply only to double bars that rotate in the same direction. As shown by Maciejewski (2008), in counter-rotating double bars, inner bars are supported by loops corresponding to a different orbital family ( $x_4$ ), and their pattern speeds may not be limited in a similar way.

## 4. Conclusions

Double bars, like single bars, can be studied with orbital analysis, but since their appearance oscillates in time, for each instantaneous shape of the system one should sample points on orbits at moments when the system takes this given shape. The method of invariant loops, proposed by Maciejewski & Sparke

(1997, 2000) and developed by Maciejewski & Athanassoula (2007, 2008) serves this purpose.

Orbital structure of the inner bar in double-barred galaxies, where both bars rotate in the same direction, strongly depends on this bar's pattern speed. At large pattern speeds loops supporting the inner bar trap large volume of phase-space, but they build inner bar that is short and round. At small pattern speeds the inner bar is longer, but it pulsates and accelerates more, and the volume of chaotic zones increases. We find no evidence of minimizing chaos at resonant coupling between the two bars.

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